Image and Data Inpainting

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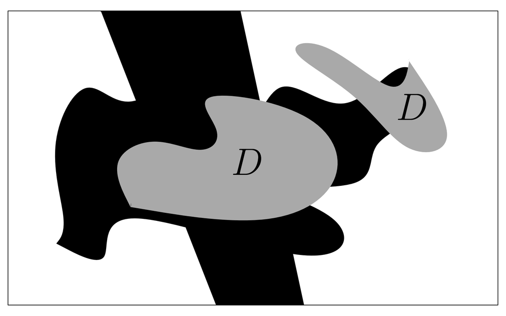
Image Inpainting

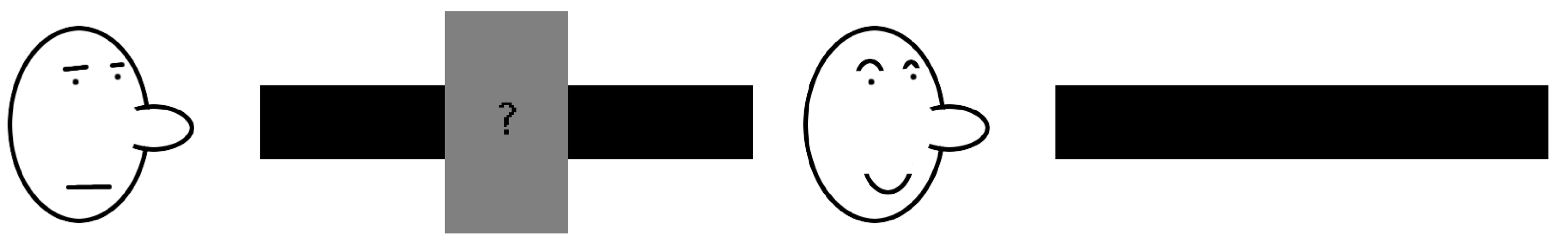
Image inpainting denotes the task of restoring a missing part of an image (inpainting domain D) in a sensible way using information from the known part.



$\Longrightarrow$

Destroyed image Inpainted image

This forms a highly ill-posed problem. Even though it is not clear what is the best way to fill in the inpainting domain, it is desirable for a method to interpolate along large gaps (*connectivity principle*):



Variational methods for inpainting

In the variational approach the target is to obtain the reconstructed image as a minimiser of a problem of the following kind:

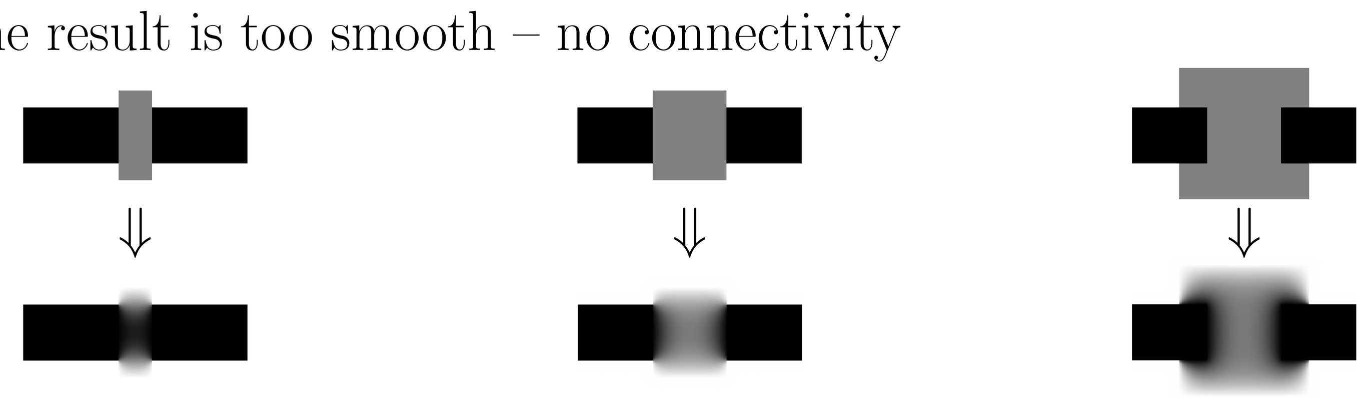
$$\min_{u\in\mathcal{B}} \underbrace{\int_{\Omega\setminus D}(u-f)^2dx}_{\text{fidelity term}} + \underbrace{\alpha\Psi(u)}_{\text{regulariser}}\,. \qquad \text{(P)}$$

Here, Ω is typically a rectangle, $D\subseteq\Omega$ is the unknown part of the data f and $\mathcal{B}$ is an appropriate space of functions with domain Ω. The parameter α balances the two terms in an optimal way. Small values of the fidelity term ensure that the intact part of the image remains intact, while the minimisation of the regulariser is responsible for the filling in process.

Different choices of Ψ result in different qualitative restorations of the missing part D.

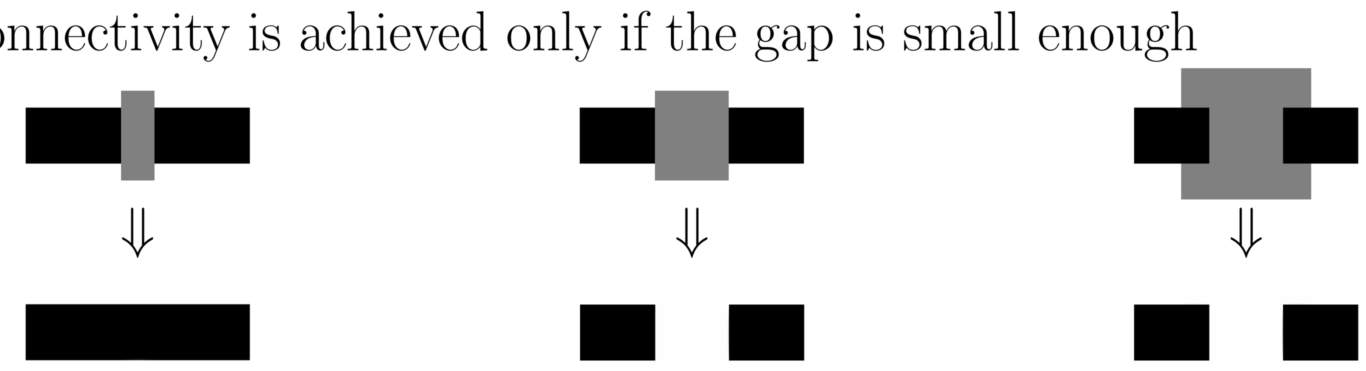
Examples

Harmonic inpainting: $\Psi(u)=\int_\Omega |\nabla u|^2 dx$
$\Rightarrow$ The result is too smooth – no connectivity



Harmonic inpainting restorations

Total variation inpainting: $\Psi(u)=TV(u)$
$\Rightarrow$ Connectivity is achieved only if the gap is small enough

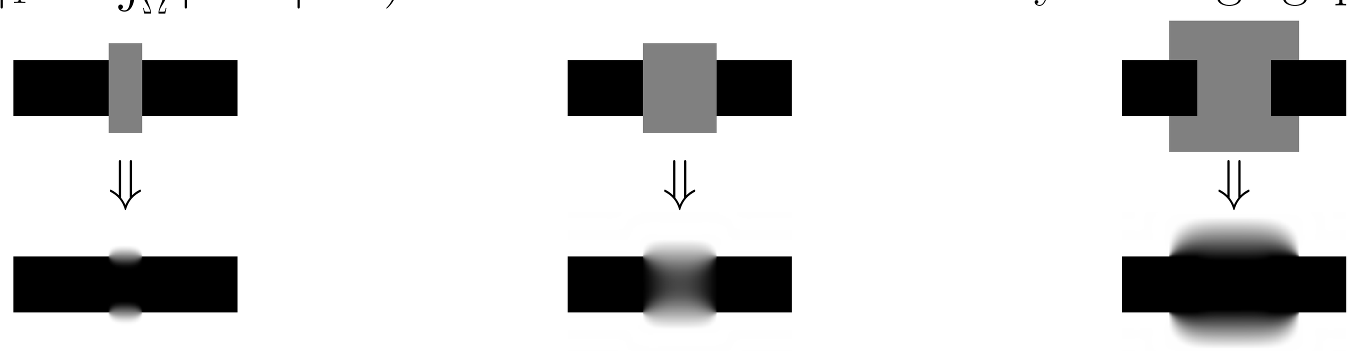


TV inpainting restorations

For smooth enough functions, the total variation can be thought as the 1-norm of the gradient, $\|\nabla u\|_1 := \int_\Omega |\nabla u|\, dx$. TV minimisation leads to *piecewise constant reconstructions.*

The TV^2 inpainting approach

Choosing as a regulariser the second order total variation TV^2 (which, for smooth enough functions, can be thought as the 1-norm of the Hessian, $\|\nabla^2 u\|_1 := \int_\Omega |\nabla^2 u|\, dx$) one can achieve connectivity for large gaps:



TV^2 inpainting restorations

TV^2 inpainting results look more realistic since they avoid the – undesirable for natural images – piecewise constant structure of TV minimisation:

Destroyed TV inpainted TV^2 inpainted

Destroyed-detail TV inpainted-detail TV^2 inpainted-detail

The numerical solution

Problem (P) is formulated as follows:

$$\min_u \|\mathcal{X}_{\Omega\setminus D}(u-f)\|_2^2 + \alpha\|\nabla^2 u\|_1.$$

This can be efficiently solved thought the *split Bregman* iteration after introducing the auxiliary variables $\tilde{u}=u$ and $w=\nabla^2 u$:

$$
\begin{aligned}
u^{k+1} &= \operatorname*{argmin}_u \ \|\mathcal{X}_{\Omega\setminus D}(u-f)\|_2^2 + \frac{\lambda_0}{2}\|b_0^k+\tilde{u}^k-u\|_2^2, && \text{Explicit solution} \quad (1)\\
\tilde{u}^{k+1} &= \operatorname*{argmin}_{\tilde{u}} \ \frac{\lambda_0}{2}\|b_0^k+\tilde{u}-u^{k+1}\|_2^2 + \frac{\lambda_1}{2}\|b_1^k+\nabla^2\tilde{u}-w^k\|_2^2, && \text{Solved with FFT} \quad (2)\\
w^{k+1} &= \operatorname*{argmin}_w \ \alpha\|w\|_1 + \frac{\lambda_1}{2}\|b_1^k+\nabla^2\tilde{u}^{k+1}-w\|_2^2, && \text{Explicit solution} \quad (3)\\
b_0^{k+1} &= b_0^k+\tilde{u}^{k+1}-u^{k+1}, && \text{Simple update} \quad (4)\\
b_1^{k+1} &= b_1^k+\nabla^2\tilde{u}^{k+1}-w^{k+1}, && \text{Simple update} \quad (5)
\end{aligned}
$$

where λ_0, λ_1 are positive parameters. This leads to a very fast method, roughly of the same order with TV minimisation, producing however more satisfactory results in general.

References

1. Papafitsoros K., Schönlieb C.B. and Sengul B., *Combined first and second order total variation inpainting using split Bregman*, Image Processing Online, 112-136, (2013), http://dx.doi.org/10.5201/ipol.2013.40. **Online demo available.**
2. Papafitsoros K. and Schönlieb C.B., *A combined first and second order variational approach for image reconstruction*, Journal of Mathematical Imaging and Vision, vol. 48, no. 2, 308-338, (2014), http://dx.doi.org/10.1007/s10851-013-0445-4.
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4. Goldstein T. and Osher S., *The split Bregman method for L1-regularized problems*, SIAM Journal on Imaging Sciences, vol. 2, no. 2, 323-343, (2009), http://dx.doi.org/10.1137/080725891.

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